Binomial Random Variable

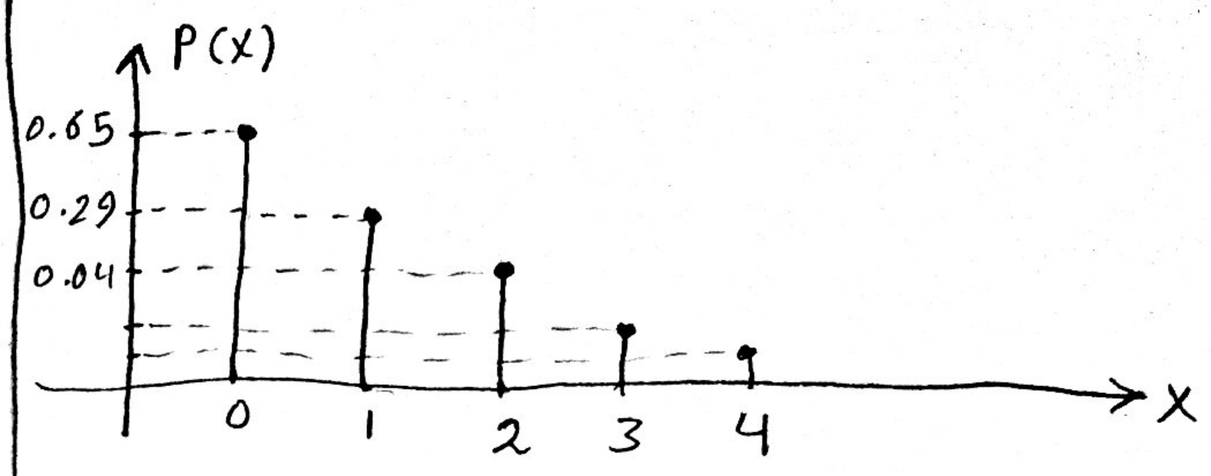
$$P(X) = \binom{n}{x} p^x q^{n-x}, \quad X = 0, 1, 2, \dots, n$$

X: is a binomial random variable that represent the no. of Succes's in n-trails.

n: No. of Trials.

P: Probability of success in a single trial.

q: (1-P) Probability of fail.



For Binomial Random Variable

$$\mu = np$$

$$Var = npq = \sigma^2$$

$$S.D = \sqrt{Var} = \sqrt{npq}$$

Ex: In an experiment 10% of People passes. Suppose that 4 persons are selected at random.

a) Find the probability that none of the Four persons passes the test.

b) _____ three persons passes.

d) Derive Formula for P(x), the probability distribution fn. of the binomial random Variable X

a) Non-passes

$$P(FFFF) = (0.9)^4$$

$X=0 \quad n=4 \quad p=0.1 \quad q=0.9$

$$P(0) = C_0^4 \, p^0 q^4 = 1 * 1 * (0.9)^4$$

b) 3-passes $\rightarrow X=3$

$$P(3) = C_3^4 \, p^3 q^1$$

$$= 4 * (0.1)^3 * (0.9)$$

d)

X	0	1	2	3	4
P(X)	0.65	0.29	0.048	0.0036	0.0001

$$P(X) = C_x^n \, p^x q^{n-x}$$

$n=4$, $P=0.1$, $q=0.9$

Sheet 5

[3] Prove that For any random Variable

i) $E(aX+b) = a E(X) + b$

$$E(aX+b) = \int_{-\infty}^{\infty} (aX+b) P(X) \, dX$$

$$= a \int_{-\infty}^{\infty} X P(X) \, dx + b \int_{-\infty}^{\infty} P(X) \, dX$$

$\int_{-\infty}^{\infty} X P(X) \, dx \rightarrow E(X)$

$\int_{-\infty}^{\infty} P(X) \, dX \rightarrow 1$

$$= a E(X) + b$$

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[7] Given a formula for P(X) for a binomial random Variable with $n = 7$ and $P = 0.2$

$$P(X) = \binom{7}{x} (0.2)^x (0.8)^{7-x}$$

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[8] Consider the Following

$$P(X) = \binom{5}{x} (0.7)^x (0.3)^{5-x}$$

a) $n = 5$ $\quad P = 0.7$

b) Graph P(X)

$P(0) =$ —

$P(1) =$ —

!

$P(5) =$ —

X	0	1	----	5
P(X)				

d) $\mu = np = 5 * 0.7$

$S.D = \sqrt{npq} = \sqrt{5 * 0.7 * 0.3}$

Report → 10

The probability of exactly k success in n repeated trials

$$b(k;n,p) = \binom{n}{k} p^k q^{n-k}$$

11 A fair coin → 6 times
head → Success

Find

i) Probability that exactly 2 heads
ii) // // at least 4 heads
iii) // // no heads
iv) // // at least one head

$n = 6$ $p = 0.5$ $q = 0.5$

i) $P(x) = \binom{n}{x} p^x q^{n-x}$

$P(2) = \binom{6}{2} (0.5)^2 (0.5)^4 =$ ✓

ii) $P(4) + P(5) + P(6)$

$= \binom{6}{4} (0.5)^4 (0.5)^2 + \binom{6}{5} (0.5)^5 (0.5)^1$
$+ \binom{6}{6} (0.5)^6 (0.5)^0 =$ ✓

ii) $P(0) = \binom{6}{6} =$ ✓

iv) $P(0) + P(1) + \cdots + P(6)$

$= 1 - b(0; 6; 0.5)$
$= 1 - P(0) =$ ✓